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A NEW APPROACH FOR IMPERFECT BOUNDARY CONDITIONS ON THE DYNAMIC BEHAVIOR

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ABSTRACT

Real beams have non-ideal boundary conditions and it is necessary to use new models to determine the real modal parameters. Models that use ideal conditions do not fully reflect reality and can lead to unsatisfactory description of the dynamic behavior. The hinged – hinged boundary conditions, which is in the focus of the paper, are not analyzed as a single beam, but as a continuous beam with three spans, free at the ends. The continuous beam with three spans is analyzed for cases in which the intermediate supports can occupy any position along the length of the beam, by an analytical solution of the problem, with the example of cases when the intermediate supports are located very close at the free ends of the continuous beam, thus simulating the real case for an hinged beam at both ends; the situation in which the intermediate supports are very close to one of the ends of the beam, thus simulating the real case of the clamped beam, with an imperfect clamped end; and the situation in which the intermediate supports are very close located anywhere on the beam length, thus simulating the hypothetic case with a continuous beam free at the ends and fix on the hinged supports. The analytic results are compared with numerical results by using finite elements method.

Keywords: natural frequency, boundary conditions, dynamic behavior

1. INTRODUCTION

Measuring natural frequencies requires the use of relatively inexpensive and very robust instruments [1, 2]. The determination of natural frequencies are easy to calculate both by using analytical models and by using numerical methods.

However, the precise calculation of the natural frequencies is significantly influenced by the correct positioning of the supports, respectively by the correct choice of the boundary conditions [3].

For example, the analytical calculation for a simple supported beam involves positioning the supports exactly at the ends of the beam, but in the real case, the positioning of these supports is very close to the ends of the beam, which requires treating the problem as a continuous beam with three openings.

Complex studies on the calculation of natural frequencies and modal shapes of continuous beams can be found in [4] and [5].

For this reason, the paper focuses on the analysis of natural frequencies and percentage deviations by applying the ideal boundary conditions and considering the imperfect boundary conditions, by moving the supports by 1%, 2% of the beam length.

2. MATERIALS AND METHODS

2.1. Analytical approach

It will be considered a continuous beam with three openings supported with two intermediate hinges and free at the ends.

The lengths of the spans are denoted (Fig. 1) with l_1, l_2 and l_3 and the sum of the lengths are equal to one, thus: $l_1 + l_2 + l_3 = 1$.

Using Euler-Bernoulli theory for each characteristic point on the beam (1, 2, 3, 4), the following boundary conditions can be written:

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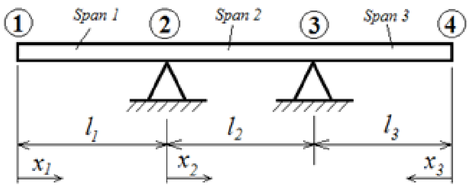

Figure 1. Continuous beam with three spans, free at the ends.

$$\langle 1\rangle \begin{cases} W_1''(0)=0 \\ W_1'''(0)=0 \end{cases} \qquad \langle 4\rangle \begin{cases} W_3''(0)=0 \\ W_3'''(0)=0 \end{cases}$$
$$\langle 2\rangle \begin{cases} W_1\ (l_1)=0 \\ W_2\ (0)=0 \\ W_1'(l_1)=W_2'(0) \\ W_1''(l_1)=W_2''(0) \end{cases} \qquad \langle 3\rangle \begin{cases} W_2\ (l_2)=0 \\ W_3\ (l_3)=0 \\ W_2'(l_2)=-W_3'(l_3) \\ W_2''(l_2)=W_3''(l_3) \end{cases} \tag{1}$$

where:
$W_i(x_i)=A_i sin(a_n x_i)+B_i cos(a_n x_i)+C_i sinh(a_n x_i)+D_i cosh(a_n x_i)$ is the normal mode shape of the span;
i = 1, 2, 3 represents the number of spans;
n is the nth vibration mode number;
A_i, B_i, C_i, D_i are the integration coefficients;
$x_1 \in [0, l_1]$, $x_2 \in [0, l_2]$, $x_3 \in [0, l_3]$;
a_n - the eigenvalues, obtained as solutions of the characteristic equation:

$$(Z_{12}\cdot Z_{21}+Z_{11}\cdot Z_{22})\cdot Z_{32}+(Z_{12}\cdot Z_{22}+Z_{11}\cdot Z_{23})\cdot Z_{31}=0 \tag{2}$$

and,

$$\begin{cases} Z_{11}=2\dfrac{1+\cos(a_n l_1)\cosh(a_n l_1)}{\cos(a_n l_1)+\cosh(a_n l_1)} \\ Z_{12}=-2\dfrac{\cos(a_n l_1)\sinh(a_n l_1)-\sin(a_n l_1)\cosh(a_n l_1)}{\cos(a_n l_1)+\cosh(a_n l_1)} \end{cases}$$
$$\begin{cases} Z_{31}=2\dfrac{1+\cos(a_n l_3)\cosh(a_n l_3)}{\cos(a_n l_3)+\cosh(a_n l_3)} \\ Z_{32}=-2\dfrac{\cos(a_n l_3)\sinh(a_n l_3)-\sin(a_n l_3)\cosh(a_n l_3)}{\cos(a_n l_3)+\cosh(a_n l_3)} \end{cases} \tag{3}$$
$$\begin{cases} Z_{21}=1-\cos(a_n l_2)\cosh(a_n l_2) \\ Z_{22}=\cos(a_n l_2)\sinh(a_n l_3)-\sin(a_n l_2)\cosh(a_n l_2) \\ Z_{23}=2\sin(a_n l_2)\sinh(a_n l_2) \end{cases}$$

By knowing the eigenvalues, the natural frequencies for the continuous beam with three openings can be calculated using the relation:

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$$f_n = \frac{a_n^2}{2\pi}\sqrt{\frac{E \cdot I}{m \cdot L^4}} \tag{4}$$

where:
f_n [Hz] is the natural frequency;
E [N/m^2] is the elasticity modulus;
I [m^4] is the moment of inertia;
m [kg] is the beam mass;
L [m] is the beam length.
In conclusion, in order to calculate the natural frequencies, we must obtain our own values with relation (2), to know the material from which the continuous beam is made and its geometry.
For this, it will be considered the beam made of steel with a density $\rho = 7800$ kg/m^3 and $E = 2.1 \cdot 10^{11}$ N/m^2, having length L =1 m.
The beam is considered to have a constant cross section, a rectangular shape with a width of b = 0,05 m and a thickness of h = 0,005 m.

2.2. Numerical approach

The validation of the results obtained by the analytical method was done using FEM analysis.
The beam described in the previous chapter was analyzed for different locations of the intermediate supports.
For the mesh of the 3D model, finite elements with an average size of 1 mm were used, and the intermediate supports are frictionless hinges located in the neutral axis of the beam.
For $l_1 = l_2 = l_3 = L/3$, the eigenvalues (a_n) obtained with relation (2) and the natural frequencies for the first $n = 6$ vibration modes calculated analytically (f_{n_a}) and obtained by FEM (f_{FEM}) are presented in Tab. 1.

Table 1. The first 6 natural frequencies calculated analytically and obtained by FEM

n	**1**	**2**	**3**	**4**	**5**	**6**
a_n	4.237	4.947	10.732	12.827	14.118	20.104
f_{n_a} [Hz]	21.401	29.168	137.295	196.111	237.594	481.771
f_{FEM} [Hz]	21.720	29.474	137.960	196.870	239.390	483.830
ε [%]	**1.492**	**1.050**	**0.484**	**0.387**	**0.756**	**0.427**

Comparing the results obtained for the natural frequencies, presented in table 1, it can be seen that the error ε [%] between the two methods is below 1.5% for vibration modes 1 and 2, respectively below 1% for the other vibration modes.

3. RESULTS

The first particular case considered is that in which the intermediate supports are located very close to the free ends of the continuous beam, thus simulating the real case of a simply supported beam, with hinges at the ends.
The analyzed cases took into account the values of natural frequencies calculated analytically, for $l_1 = l_3 =$ 0.02 m; l_1 = 0.01 m, l_3 = 0.02 m; $l_1 = l_3$ = 0.01 m and their comparison with the natural frequencies obtained for the case of the simply supported beam ($L = 1$) and percentage deviations from them.
The results are presented in Tab. 2
The percentage deviations are shown in Fig. 2.

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Table 2. Natural frequencies and percentage deviations for a simple supported beam with imperfect boundary conditions

	n	**1**	**2**	**3**	**4**	**5**	**6**
L = 1 m	**f_{n_a} [Hz]**	**11.764**	**47.057**	**105.878**	**188.227**	**294.104**	**423.510**
$l_1 = 0.02$	f_{n_a} [Hz]	12.764	51.048	114.823	204.044	318.647	458.546
$l_3 = 0.02$	**ε [%]**	**8.500**	**8.481**	**8.449**	**8.403**	**8.345**	**8.273**
$l_1 = 0.01$	f_{n_a} [Hz]	12.503	50.006	112.495	199.946	312.323	449.582
$l_3 = 0.02$	**ε [%]**	**6.278**	**6.267**	**6.250**	**6.226**	**6.195**	**6.156**
$l_1 = 0.01$	f_{n_a} [Hz]	12.249	48.996	110.236	195.966	306.178	440.862
$l_3 = 0.01$	**ε [%]**	**4.123**	**4.120**	**4.117**	**4.112**	**4.105**	**4.097**

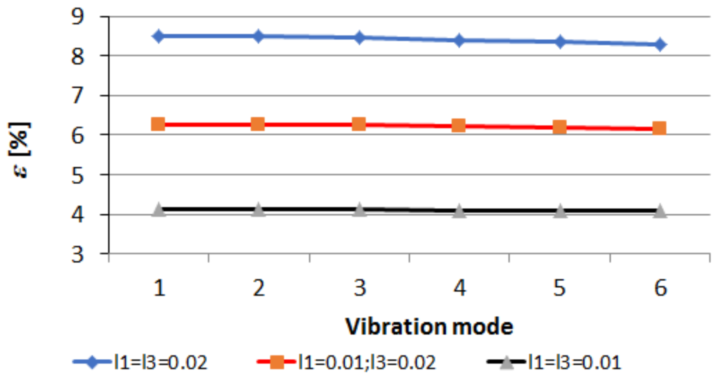

Figure 2. Percentage deviations for the case of the simply supported beam with imperfect boundary conditions.

The second particular case considered is that in which the intermediate supports are located very close to the left free end of the continuous beam, thus simulating the case of a imperfect clamped – free beam.
The analyzed cases took into account the values of natural frequencies calculated both analytically and obtained by FEM, for $l_1 = l_2 = 0.02$ m;$l_1 = 0.02$ m, $l_2 = 0.01$ m; $l_1 = 0.01$ m, $l_2 = 0.02$ m; $l_1 = l_2 = 0.01$ m and their comparison with the natural frequencies obtained for the ideal case ($L = 1$) of the clamped– free beam.
The results are presented in Table 3and the percentage deviations are shown in figure 3.

Table 3. Natural frequencies and percentage deviations for a clamped - free beam with imperfect boundary conditions

	n	**1**	**2**	**3**	**4**	**5**	**6**
L = 1 m	**f_{n_a} [Hz]**	**4.191**	**26.264**	**73.541**	**144.110**	**238.225**	**355.866**
$l_1 = 0.02$	f_{n_a} [Hz]	4.486	28.119	78.757	154.374	255.258	381.408
$l_2 = 0.02$	**ε [%]**	**7.029**	**7.063**	**7.093**	**7.122**	**7.150**	**7.177**
$l_1 = 0.02$	f_{n_a} [Hz]	4.424	27.726	77.639	152.153	251.537	375.779
$l_2 = 0.01$	**ε [%]**	**5.558**	**5.566**	**5.574**	**5.581**	**5.588**	**5.595**
$l_1 = 0.01$	f_{n_a} [Hz]	4.394	27.546	77.151	151.226	250.052	373.628
$l_2 = 0.02$	**ε [%]**	**4.849**	**4.881**	**4.909**	**4.937**	**4.965**	**4.991**
$l_1 = 0.01$	f_{n_a} [Hz]	4.334	27.165	76.068	149.074	246.446	368.172
$l_2 = 0.01$	**ε [%]**	**3.422**	**3.430**	**3.437**	**3.444**	**3.451**	**3.458**

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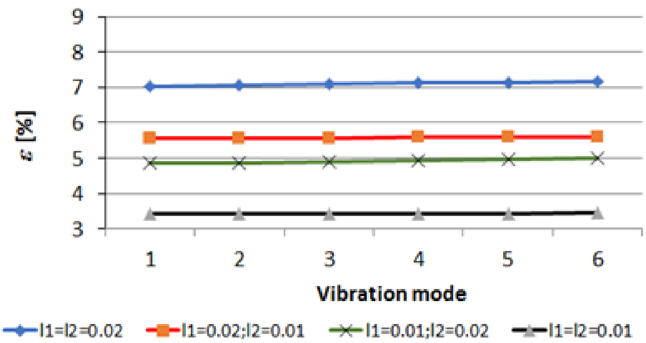

Figure 3. Percentage deviations for the case of the clamped-free beam with imperfect boundary conditions.

The third particular case considered is that in which the intermediate supports are located very close on the continuous beam, thus simulating the hypothetic case of a free – clamped – free continuous beam.
The analyzed cases took into account the values of natural frequencies calculated both analytically and obtained by FEM, for l_1 = 0.20 m and l_2 = 0.02 m.
The results are presented in Tab. 4.

Table 4. Natural frequencies and percentage deviations when the intermediate supports are very close on the continuous beam

	n	**1**	**2**	**3**	**4**	**5**	**6**
l_1 = 0.20 l_2 = 0.02	a_n	2.384	5.969	9.086	9.997	13.991	17.990
	f_{n_a} [Hz]	6.774	42.465	98.393	119.128	233.320	385.751
	f_{FEM} [Hz]	6.796	42.589	99.571	119.400	233.930	386.840
	ε [%]	**0.329**	**0.293**	**1.197**	**0.229**	**0.262**	**0.282**

A comparison of the vibration modes between the analytical method and the FEM method, for l_1= 0.20 m andl_2 = 0.02 m, is illustrated in Fig. 4.

4. CONCLUSIONS

In this paper we have applied three cases for which imperfect contour conditions generate errors in the calculation of natural frequencies for simple structures.
Tab. 1 and 4 show that, regardless of the method applied, the analytical method and the numerical method, respectively, when calculating the natural frequencies, approximately the same values are obtained for the continuous beam with three openings. The errors obtained by the two calculation methods are below 1%, except for modes 1 and 2 for the case when the intermediate supports are positioned equidistant from the ends of the bar, respectively for mode three (Tab 4) in case of simulating the imperfect stiffness, but for this, in this case, the shape of the vibration modes obtained by the two methods has the same shape.
In the case of simply supported beams, a displacement of the supports towards the ends of the beam by 2% produces deviations of the natural frequencies of more than 8% from the case when the supports are positioned at the ends of the beam,the precision deviation decreases to about 4% for the displacement of the supports by 1% with respect to the ends of the beam.
In the case of the imperfectly clamped-free beam (Tab. 3), the positioning of the intermediate supports with 2% - 4% produces deviations in the calculation of the own frequencies of 3% - 7%.
Regardless of the analyzed cases, for the first 6 analyzed vibration modes, the deviations have approximately the same value (Fig. 2 and 3).

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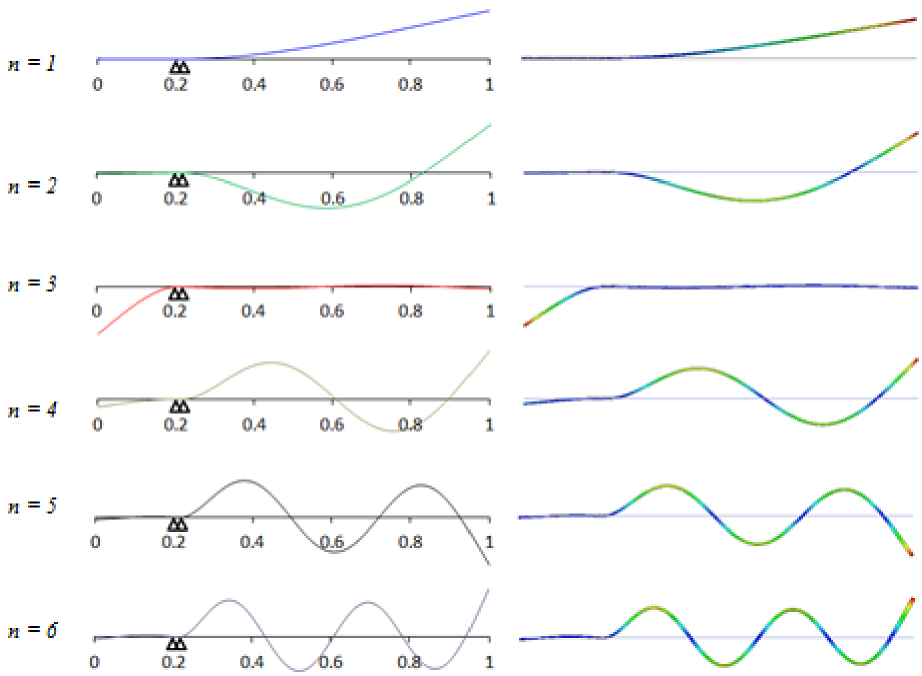

Figure 4. A comparison for the six vibration modes for l_1 = 0.20 and l_2 = 0.02 obtained by analytical (left) and numerical method (right).

In conclusion, for real cases, small deviations of the position of the beam supports from the ends, or an imperfect fixation of the beam lead to significant deviations of the natural frequencies. It is important that, when calculating the natural frequencies for real beams, we take into account the imperfections of the positioning of the supports.

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